Mid-term exam for Quantum Physics 1 - 2015-2016
Thursday 24 September 2015, 9:00 - 10:00

READ THIS FIRST:

- Clearly write your name and student number on each answer sheet that you use.
- On the first answer sheet, write the total number of answer sheets that you turn in.
- Start each question (number T1, T2) on a new side of an answer sheet.
- The course is in English, please answer in English as much as you can.
- Note that this test has 2 questions, it continues on the backside of the paper!
- The test is open book within limits. You are allowed to use the book by Griffiths, the copies from the Feynman book, and one A4 sheet with personal notes, but nothing more than this.
- If you get stuck on some part of a problem for a long time, it may be wise to skip it and try the next part of a problem first. The test is only 1 hour.
- When you turn in your problems, please **put your answer sheets in the order T1, T2** and **staple** them together.

Useful formulas and constants:

Electron mass	m_e	$= 9.1 \cdot 10^{-31}$ kg
Electron charge	$-e$	$= -1.6 \cdot 10^{-19}$ C
Planck's constant	h	$= 6.626 \cdot 10^{-34}$ Js $= 4.136 \cdot 10^{-15}$ eVs
Planck's reduced constant	$\hbar$	$= 1.055 \cdot 10^{-34}$ Js $= 6.582 \cdot 10^{-16}$ eVs

Fourier transform relations between x- and k-representation of a state:

$$\overline{\Psi}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-ikx}\, \Psi(x)\, dx$$

$$\Psi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{ikx}\, \overline{\Psi}(k) dk$$

Problem T1 - ***For this problem, you must write up your answers in Dirac notation.***

Consider a quantum system that has two energy eigenstates, as characterized by the following equations:

$$\hat{H}|\varphi_1\rangle = E_1|\varphi_1\rangle$$
$$\hat{H}|\varphi_2\rangle = E_2|\varphi_2\rangle$$

where $0 < E_1 < E_2$ the two energy eigenvalues, and $|\varphi_1\rangle$ and $|\varphi_2\rangle$ two orthogonal, normalized energy eigenvectors. The observable $\hat{A}$ is associated with the electric dipole A of this quantum system. For this system,

$$\langle\varphi_1|\hat{A}|\varphi_1\rangle = 0 \quad , \quad \langle\varphi_2|\hat{A}|\varphi_2\rangle = 0 \quad , \quad \langle\varphi_1|\hat{A}|\varphi_2\rangle = \langle\varphi_2|\hat{A}|\varphi_1\rangle = A_0 \quad \text{(here } A_0 > 0 \text{ and real).}$$

Note that the states $|\varphi_1\rangle$ and $|\varphi_2\rangle$ are energy eigenvectors, and that they are ***not*** eigenvectors of $\hat{A}$.

a) [2 points]

At some time defined as $t = 0$, the system is in a state that can be described as (with all c_n a complex-valued constant)

$$|\Psi_0\rangle = c_1\,|\varphi_1\rangle + c_2\,|\varphi_2\rangle = \sqrt{5}\,|\varphi_1\rangle + e^{i\varphi}\sqrt{10}\,|\varphi_2\rangle \ .$$

Here $\varphi = 2$ is a phase for one of the probability amplitudes for the state at $t = 0$. Note, however, that this description of the state is not normalized. Write down a description of this state of the system that is normalized.

b) [3 points]
Now assume that at time $t = 0$ the system is in the state

$$|\Psi_2\rangle = e^{i\alpha}\sqrt{\tfrac{1}{2}}\,|\varphi_1\rangle + \sqrt{\tfrac{1}{2}}\,|\varphi_2\rangle .$$

Here $\alpha = \pi$ is a phase for one of the probability amplitudes for this state at $t = 0$. Write down an expression that describes for this case the state of the system at times $t > 0$. Work out your answer such that your expression does not contain operators. Your expression can contain constants (if you need them) such as E_1, E_2, A_0, etc.

c) [5 points]
For the same initial state as in question **b)**, make a graph of the expectation value for $\langle\hat{A}\rangle$ as a function of time, for the time interval $t = 0$ ns till $t = 20$ ns. Use for your graph these values for the mentioned constants: $A_0 = 1$ e·nm (where e is $1.6 \cdot 10^{-19}$ C), $E_1 = 6.626 \cdot 10^{-26}$ J, and $E_2 = 2\,E_1$.
As a first step, write down an expression that describes how the expectation value for $\langle\hat{A}\rangle$ changes as a function of time, for $t > 0$.

Problem T2
a) [4 points]
Consider a quantum particle with mass m that can only move in one dimension (x-direction). At some moment the state of the particle can be described with a wave function that is a function of the particle's wavenumber k, and which is $\overline{\Psi}(k) = 1/\sqrt{a}$ for $|k| < a/2$ and zero elsewhere (here k is the wavenumber = 2π/wavelength).
Write down a wave function that describes the position x of this particle, for the case that the particle is in the state $\overline{\Psi}(k)$. In your answer, work out the expressions and algebra as far as possible.

b) [4 points]
Assume that the particle is in the state of question **a)**. What is the expectation value for the position for the particle? If possible, give both the expression that you need to use for the calculation and the actual value of the expectation value (and explain your answer if it is not based on a full calculation).

c) [3 points]
Assume that the particle is in the state of question **a)**. The value for a in the description of the state is $a = 1 \cdot 10^{10}$ m^{-1}. The mass of the particle is $m = 1 \cdot 10^{-30}$ kg.
What is the maximum value for the momentum p_x of the particle that you can obtain if you would measure its value?
(If you already know about the difference, assume that you can simply work with phase velocity, rather than using the group velocity.)